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Comparison of heuristics for an economic lot scheduling problem with deliberated coproduction

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Abstract: We built on the Economic Lot Scheduling Problem Scheduling (ELSP) literature by making some modifications in order to introduce new constraints which had not been thoroughly studied with a view to simulating specific real situations. Specifically, our aim is to propose and simulate different scheduling policies for a new ELSP variant: Deliberated Coproduction. This problem comprises a product system in an ELSP environment in which we may choose if more than one product can be produced on the machine at a given time. We expressly consider the option of coproducing two products whose demand is not substitutable. In order to draw conclusions, a simulation model and its results were developed in the article by employing modified Bomberger data which include two items that could be produced simultaneously.

Keywords: ELSP, coproduction, simulation, heuristics

1 Introduction

This paper considers a variation on the economic lot scheduling problem (ELSP). The ELSP is concerned with lot sizing and scheduling the production of several different items on a single machine. The objective of ELSP is to determine lot sizes and a production schedule so that the sum of inventory holding costs and set-up costs is minimized. The problem is characterized by the following: no more than one product can be produced on the machine at a given time, production rates are

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deterministic and constant, product set-up costs and times are independent of production order, product demand rates are deterministic and constant, demand must be met in the periods in which it occurs, inventory costs are directly proportional to inventory levels, and production capacity is sufficient to meet the total demand.

According to Boctor (1987), the problem could occur in many situations, such as molding and stamping operations, bottling, metal forming, and plastic production lines (press lines, plastic and metal extrusion machines), weaving production lines (textiles, carpets), paper production, etc. In practical situations however, some characteristics of the classical ELSP are commonly modified (Vidal-Carreras & Garcia-Sabater, 2005; Vidal-Carreras, Garcia-Sabater, Marin-Garcia, & Garcia-Sabater, 2008). So in this paper, we aim to propose and simulate different scheduling policies to a new ELSP variant: Coproduction ELSP. We may state that coproduction appears in a product system in which more than one product can be produced on the machine at a given time (Deuermeyer & Pierskalla, 1978). This paper specifically considers the situations of deliberated coproduction of two products. So this problem consists in a product system in an ELSP environment where two products can be produced on the machine at a given time. It is important to note that we are working on a problem with which we can choose when we wish to coproduce, or not, hence its name: deliberated coproduction. Those products which are produced jointly (coproducts) have their own market, so demand is not substitutable. The products resulting from the process are not ordered according to any kind of hierarchy.

In the next sections, we review the related literature, summarize the logic of each heuristic employed and how all the heuristics were implemented for our simulation tests, and we finally present our results which are discussed in order to draw conclusions.

2 Literature review

The economic lot scheduling problem (ELSP) has been studied in the literature for approximately 50 years (Rogers, 1958; Eilon, 1957). A comprehensive review of the ELSP until the late seventies can be found in Elmaghraby (1978), who divides approaches into two categories; analytical approaches that achieve the optimum for a restricted version of the original problem; heuristic approaches that achieve

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''good'' solutions for the original problem. More recent review studies can be found in Karimi, Ghomi, and Wilson (2003), Lopez and Kingsman (1991), Sox, Jackson, Bowman, and Muckstadt (1999) and Zhu and Wilhelm (2006). Because of its nonlinearity, combinatorial characteristics and complexity, the ELSP is generally known as an NP-hard problem (Hsu, 1983; Gallego & Shaw, 1997).

Typical productive areas in which coproduction appears are the electronic industry (semiconductors, diodes, transistors) (Bitran & Gilbert, 1994), the petrochemical industry (Lisbona & Romeo, 2008) and the glass industry (Oner & Bilgic, 2008). All these industries use common processes in which quality and specifications can lead to diversified products. Deuermeyer and Pierskalla (1978) propose an optimal control model to minimize the costs of production, inventory holding and backorders in a two-product system for which two production processes are available. These authors show that it may be optimal to produce items jointly or separately depending on the current inventory positions. Ou and Wein (1995) examine a case in which a family of products is serially ordered in terms of quality. There is a process for each product which yields both that product and those of lower quality as by-products. Yields are assumed to be random. These authors derive scheduling policies from the exact solution to a Brownian motion control model of the production and inventory system. Bitran and Dasu (1992) consider a situation in which a single process has an output with different grades of quality in which higher grades of output can be used to satisfy the demands for the lower grades of the output. The fraction of output is random. The objective is to maximize the expected profit. Bitran and Gilbert (1994) study a version of the same problem but whose objective is to minimize the expected cost which comprises production, inventory holding, and shortage costs. Alternative lot sizing policies are considered, which range from simple ones to those that consider the impact of downgrading and production smoothing under different downgrading policies. All these authors consider that demand for products is substitutable or can be transformed into a structure where substitutions of demand are transitive. Oner and Bilgic (2008) consider a problem in the ELSP environment which includes coproduction in which demands are not substitutable. In this paper, the authors propose a model to include uncontrolled coproduction in ELSP. However this model, which is based on the Common Cycle approach, does not work suitably when the ratio of set-up costs to holding costs between products is not equal (Jones & Inman, 1989).

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Testing heuristics through simulation is another topic covered in the literature. Specifically, we focus on the references which modify simple heuristics for the classical ELSP in order to add the new constraints which appear in practical situations. Specifically, production systems with dynamic stochastic demand (Leachman & Gascon, 1988; Gascon, Leachman, & Lefrancois, 1994), static stochastic demand (Vergin & Lee, 1978; Brander, Leven, & Segerstedt, 2005) and hybrid make-to-order and make-to-stock systems (Soman, Pieter van Donk, & Gaalman, 2004; Leachman et al., 1988; Gascon et al., 1994) have been simulated. In some cases the same heuristics have been tested with modifications in the input conditions. Vergin et al. (1978) were the first group to propose and test dynamic scheduling policies based on feedback and inventory levels under varying cost and system parameters. They tested two rules for deterministic demand: classical cyclical production lot size for multiple products (EOQ), modified EOQ to incorporate shortages costs and four rules for dynamic scheduling: Magee's Rule and three alterations of this rule which incorporate maximum inventory level, backorders and the elimination of a very short production run. Leachman et al. (1988) tested four rules for five products with dynamic stochastic demand on a single machine. The rules are the following: a dynamic length heuristic proposed by them in this article, a policy based on independent economic manufacturing quantities for each item, a policy based on the Doll and Whybark procedure, and a policy utilizing the Vergin et al. (1978) dynamic scheduling rules, involving five items produced on a single machine. In Gascon et al. (1994), six different heuristics for five items with stationary demand, and with and without forecast errors and dynamic demand, are tested. They compare: the Vergin and Lee policy, the look-ahead heuristic of Gascon, the dynamic cycle lengths heuristic (Leachman et al., 1988) and the enhanced dynamic cycle lengths heuristic (Leachman, Xiong, Gascon, & Park, 1991) approaches with two simpler rules: one based on independent economic production quantity and the other based on the Doll and Whybark procedure. Soman et al. (2004) tested four dynamic scheduling policies with modified Bomberger data that include the conditions of hybrid MTO and MTS products. Finally in Brander, Leven and Segerstedt (2005), we find a simulation study that employs a dynamic programming approach from Bomberger and a heuristic method from Segerstedt to calculate lot sizes for four items with stationary stochastic demand. We could generalize that all these authors finish their studies with two main conclusions: the policies which consider current inventory levels and appropriate decision rules in making scheduling decisions

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outperform policies based solely on the solutions of an ELSP (deterministic) model, and the methods that perform well for classic ELSP conditions do not necessarily perform well for ELSP variants.

So we can conclude this section by stating that our study differs from the literature in several aspects. We tested different heuristics with a new ELSP variant: deliberated coproduction. This kind of coproduction is especially interesting because it is controlled and there is no hierarchy among the resulting products, so each product has its own market.

3 Problem description

We consider a problem of scheduling items when two of them can be produced at the same time in the single facility with a limited capacity. This problem is named ELSP with Coproduction. The objective is to minimize total costs $\left(\sum C_i\right)$ by determining the optimal T_i, T_{ij} subject to the capacity requirement constraint. We use the following notations:

i,j	Index of products, i,j=1...N
d_i	Demand rate in units per time for product i
h_i	Inventory carrying cost per unit and time for product i
A_i, A_{ij}	Cost of set-up per product lot for product i and for products i and j when they are coproduced
c_i, c_{ij}	Time of set-up per product lot for product i and for products i and j when they are coproduced
p_i, p_{ij}	Production rate for product i and for products i and j when they are coproduced
T_i, T_{ij}	Cycle time for item i, and for products i and j when they are coproduced
H	Total annual number of production days of capacity available

The total costs equation for the economic manufacturing quantity incorporating coproduction considers that product i can be produced with or without product j:

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$$\text{Total Cost} = \sum_i C_i + \sum_{ij} C_{ij}$$

where,

$$\left\{ \begin{array}{l} C_i = \frac{H}{T_i} A_i + h_i \frac{T_i}{2} d_i \left(1 - \frac{d_i}{p_i}\right) \\ C_{ij} = \frac{H}{T_{ij}} A_{ij} + h_i \left[\frac{T_{ij}}{2} d_i \left(1 - \frac{d_i}{p_{ij}}\right) + \frac{T_{ij}}{2} d_j \left(1 - \frac{d_j}{p_{ji}}\right) \right] \end{array} \right\}$$

Equation 1. "Total Costs for the Economic Manufacturing Quantity with Coproduction".

We make the following assumptions in this paper:

- One or two products, i or i+j, can be produced on the machine at a given time
- Product demand rates are stochastic with mean d_i
- Product production rates are deterministic and constant
- Product set-up costs and times are independent of production order
- Inventory costs are directly proportional to inventory levels
- Production capacity is sufficient to meet the total demand

4 Scheduling rules

In this section, we present a brief summary of the modified various scheduling rules for the purpose of including coproduction. These heuristics are: EMQ, Doll & Whybark (1973) and Fransoo (1993). The rationale for including these simple heuristics in the comparison is to obtain a better understanding of the value of added coproduction in the scheduling rules. These methods are basically run-out-based scheduling rules, which are widely used in industry as they are easy to understand and implement.

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4.1 Preliminary concepts

In order to apply heuristics correctly, we must define the values of the initial and safety stocks of each item. On the one hand, we consider that initial inventories are equal to half the maximum stock for all the heuristics. On the other hand, safety stock levels are determined by deploying the standard textbook method (Silver, Pyke, & Peterson, 1998) which uses demand variance and the desired service levels. So for service levels of 95%, we can determine the safety stock by Equation 2:

$$ss_i = 1.65\sigma\sqrt{T_i\left(1 - d_i/p_i\right)}$$

Equation 2. "Safety Stocks". Source: Silver et al. (1998).

in which formula σ is the standard deviation of demand, and T_i is the target cycle according to the corresponding heuristic. By completing the adaptation to be able to incorporate coproduction in a production cycle, we define safety stocks for product i when it is to be produced with product j according to Equation 3, where T_{ij} is the target cycle when products i and j are coproduced:

$$ss_i^* = 1.65\sigma\sqrt{T_{ij}\left(1 - d_i/p_{ij}\right)}$$

Equation 3. "Safety stocks with Coproduction".

We also assume that at the production decision moment, the run-out time, the RO_i for each item, is calculated. According to Gascon et al. (1994), RO_i is defined as the expected duration until the inventory of item i, named I_i, falls to a reorder point equal to the safety stocks, ss_i, plus the expected demand, d_i, during the changeover time, named c_i or c_{ij} if coproduction is done. So, RO_i is given according to Equation 4:

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$$RO_i = \begin{cases} \text{if coproduction "i + j" is not produced} \longrightarrow \dfrac{I_i - ss_i}{d_i} - c_i \\ \text{if coproduction "i + j" is produced} \longrightarrow \dfrac{I_i - ss_i^*}{d_i} - c_{ij} \end{cases}$$

Equation 4. “Run Out”. Source: Modified by Soman et al. (2004).

Without loss of generality, items are renumbered so that: $RO_1 \le RO_2 \le .... \le RO_n$. The first product is then chosen as the product to be next produced.

4.2 EMQ heuristics modified with Coproduction

The EMQ heuristics is based on the cycles for independent manufacturing $Ti = \sqrt{2A_iH \Big/ h_id_i\left(1 - d_i/p_i\right)}$ $i = 1...n$, which we modified to incorporate coproduction, as shown in Equation 5;

$$Tij = \sqrt{2A_{ij}H \Big/ \left(h_id_i\left(1 - d_i/p_{ij}\right) + h_jd_j\left(1 - d_i/p_{ij}\right)\right)} \qquad i = 1...n$$

Equation 5. “Cycle Time for the Economic Manufacturing Quantity with Coproduction”.

where H is the total annual number of production days of capacity available, and for item i=1,..,n, T_i is the target cycle, A_i, A_{ij} are the costs to set-up the process for one lot (batch) of product i, of product i with product, p_i, p_{ij} are the daily production rates of product i, or of product i produced with product j, h_i is the cost of holding one unit in inventory for one year, and d_i is the daily demand for product i. In this heuristic model, items are produced according to their economic manufacturing quantities, although the truncating production runs wherever the inventory of another item is running out. So, it is basically a multi-item (s,S) policy where ss_i is the safety stock, according to Equation 6:

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$$s_i = \begin{cases} \text{if coproduction "i + j" is not produced} \longrightarrow Smin_i = ss_i + c_i d_i \\ \text{if coproduction "i + j" is produced} \longrightarrow Smin_i^* = ss_i^* + c_i d_i \end{cases}$$

$$S_i = \begin{cases} \text{if coproduction "i + j" is not produced} \longrightarrow Smax_i = ss_i + T_i d_i \left(1 - d_i / p_i\right) \\ \text{if coproduction "i + j" is produced} \longrightarrow Smax_i^* = ss_i^* + T_{ij} d_i \left(1 - d_i / p_{ij}\right) \end{cases}$$

Equation 6. "Min Stock and Max Stock for the Economic Manufacturing Quantity with Coproduction".

So according to this rule, the production of the current item i continue until the inventory of that product reaches S_i or the inventory of another product j falls below s_j.

4.3 Doll and Whybark

Our implementation of the dynamics of Doll and Whybark's heuristics is relatively similar to the EMQ heuristics, except that it changes the way of calculating the target cycle, in our case T_i and T_{ij}. We implemented a modified version for this rule that incorporates coproduction. In Doll and Whybark's heuristics, the target cycle for item i, T_i, is a multiple of a fundamental target cycle length T, that is $T_i = k_i T$, where k_i is a positive integer. So by incorporating part of the group, we have to also consider $T_{ij} = k_i T$. The objective is to find the values of T and k_i that minimize the sum of the changeover and inventory costs for each item, i.e., incorporating coproduction:

$$\text{Min} \sum_i C_i = \sum_i \left\{ \frac{H}{k_i T} s_i + \frac{H}{k_{ij} T} s_{ij} + h_i \left[\frac{k_i T}{2} d_i \left(1 - \frac{d_i}{p_i}\right) + \frac{k_{ij} T}{2} d_i \left(1 - \frac{d_i}{p_{ij}}\right) \right] \right\}$$

Equation 7. "Total Costs for the Economic Manufacturing Quantity with Coproduction".

So, the basic period T is calculated. For this purpose, T_i and T_{ij} are calculated according to Equation 5 for each item, and T is selected as the smallest value of these, i.e., $T = \min\left\{T_i, T_{ij}\right\}$. Then, the k_i and k_{ij} values are selected as the closest

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power-of-two integer multiple (rounded up or down) to T_i/T, and T_{ij}/T that incurs less value for Function C_i. At this point, the basic period time T is recalculated using the new estimates of k_i, according to Equation 8:

$$T = \sqrt{2H\left(\sum_i s_i + \sum_{ij} s_{ij}\right) \Big/ \left(\sum_i h_i d_i \left(1 - d_i/p_i\right) + \sum_{i,j} h_i d_i \left(1 - d_i/p_{ij}\right)\right)}$$

Equation 8. “Total Cycle for the Economic Manufacturing Quantity with Coproduction”.

With this value of T, k_i estimations are recalculated. The procedure terminates when consecutive iterations produce identical values of k_i. Then, values of T_i are calculated for each item i as $T_i = k_i T$ and $T_{ij} = k_i T$.

4.4 Fransoo

Fransoo (1993) suggests a simple policy which aims to achieve stable cycle times. The idea is to stick to target cycle times as much as possible. So, the production quantity of the product chosen for production is not affected by the fact that some other product may run out. Should there be a case of high utilization, this may save the number of set-ups, hence the productive capacity. However, some orders may be lost at the same time. Based on the run-out times, product i with $\min RO_i$ is indexed as 1 and selected for production. So when the production quantity reaches $Smax_i$, it is given as Equation 9:

$$S_i = \begin{cases} \text{if coproduction "i + j" is not produced} \longrightarrow Smax_i = ss_i + T_i d_i \left(1 - d_i/p_i\right) \\ \text{if coproduction "i + j" is produced} \longrightarrow Smax_i^* = ss_i^* + T_{ij} d_i \left(1 - d_i/p_{ij}\right) \end{cases}$$

Equation 9. “Max Stock for the Economic Manufacturing Quantity with Coproduction”.

with T_i and T_{ij}, which are calculated according to our modified version of Doll et al. (1973).

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5 Simulation model

A simulation model was developed using Anylogic 6.0 to evaluate the performance of Coproduction under different scheduling heuristics. The model has two main modules: an order generator module that generates the orders based on the demand distribution, and a shop floor control module that contains the shop configuration under study and the various scheduling rules to operate the shop.

5.1 Model dynamics

"Target cycle'' times are pre-calculated using either (a) the modified EMQ incorporating coproduction, or (b) Doll and Whybark's heuristics which was also modified by incorporating coproduction.

The values obtained are shown in Table 1:

Productive Option	Target Cycle	
	EMQ Modified	Doll & Whybark Modified
1	167.53	176.11
2	37.73	22.01
3	39.26	44.03
2+3	29.24	22.01
4	19.53	11.01
5	49.68	22.01
6	106.61	44.03
5+6	135.85	88.06
7	204.33	88.06
8	20.52	22.01
9	61.48	44.03
8+9	63.38	44.03
10	39.26	44.03

Table 1. "Target Cycle modified by incorporating Coproduction".

In order to decide for coproducts 2, 3, 5, 6, 8 and 9 what quantity of item should be produced alone or jointly, we define a strip based on its stock. We consider the lines in the coproduct stock where q_i is the actual value of the stock of i, according to Table 2.

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The stock strip is an input of the problem because coproduction is deliberated. By changing the values of each coproduct's stock strip, we can create a lot of different simulation scenarios to obtain the best solution.

Stock Strip	**Product j is produced with product i if** $q_j \in$
A - narrow	$\left[0, ss_j^*\right]$
B - medium	$\left[0, \left(Smax_j^* + ss_j^*\right)/2\right]$
C- wide	$\left[0{,}3\left(Smax_j^* + ss_j^*\right)/2\right]$
D- very wide	$\left[0, Smax_j^*\right]$

Table 2. "Definition of Stock Strip".

For example, should the input be as follows:

$$\begin{cases} \textbf{Coproduction i+j} = \text{YES} \\ \textbf{Stock Strip i+j} = \text{C} \end{cases}$$

This means that the system allows the coproduction of the production of i+j, with these conditions:

- If product i is the next to be produced, it will be produced with j, if the actual stock of j q_j belongs to $\left[0{,}3\left(Smax_j^* + ss_j^*\right)/2\right]$.
- If product j is the next to be produced, it will be produced with i, if the actual stock of i q_i belongs to $\left[0{,}3\left(Smax_i^* + ss_i^*\right)/2\right]$.

Safety stock with and without coproduction (ss_i, ss_i^*) and order up-to levels for each product i with and without coproduction ($Smax_i$,$Smax_i^*$) are pre-calculated based on the mean and standard deviation of the demand during the replenishment lead-time, the desired service level, and the productive option. Initial stocks are considered according to the Table 3. These target cycle times, safety and initial stocks, and order up-to levels are used as inputs at the operational decision level.

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The timing sequence in the simulation model is as follows.

- The demand for each item is generated at the beginning of each period. Demand is fulfilled from the stock. The inventory balance is updated. If demand cannot be met, it is lost. Besides, a lost sales cost is also incurred which is proportional to the units lost and the cost per unit item. The productive option is chosen according to the stock strip and the stock levels of the coproducts.

- At the end of each production run, the run-out times are calculated for all the products and that with the shortest run-out time is selected for the next production run.

- Production start times and production quantities are calculated based on the heuristic scheduling rule chosen.

We consider a period to be a day. For each scheduling heuristics, a simulation run lasting 240 periods is performed.

5.2 Experimental conditions

All the simulations are run on a year horizon and by assuming that ten items are produced on a single machine. Production activity is assumed to be 240 days in a year, only on weekdays. To evaluate and compare the scheduling rules discussed in the earlier section, we use the Bomberger dataset which is the most commonly used in the ELSP literature (e.g., Haessler, 1979). These data are modified to incorporate coproduction, as Table 3 shows.

We decided to incorporate coproduction into products 2 and 3 because they are the first whose values are all different. Products 8 and 9 are chosen because of their symmetry with products 2 and 3. Finally, products 5 and 6 are chosen because there are in the middle of 2+3 and 8+9. We consider this small number of groups to examine the effect of the coproduction phenomenon. The values of the parameters used for coproduction are needed and created according to these rules. We consider that set-up costs, set-up time and product rate $\left(A_{ij}, c_{ij}, p_{ij}\right)$ are reduced when items are produced simultaneously. Specifically, we assume that they are half the value when just produced:

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$$A_{ij} = A_{ji} = \left(A_i + A_j\right)\big/2,\ c_{ij} = c_{ji} = \left(c_i + c_j\right)\big/2,\ p_{ij} = p_i\big/2,\ p_{ji} = p_j\big/2$$

Productive Option	Part N. Bomberger		Setup Cost	Unit Cost*		Prod Rate (unit /day)		Demand** (unit/day)		Setup Time (hours)	Initial Stocks
1	1		15	0.065		30000		400		1	2200
2	2		20	0.1775		8000		400		1	1200
3	3		30	0.1275		9500		800		2	2200
2+3	2	3	25	0.1775	0.1275	4000	4750	400	800	1,5	
4	4		10	0.1		7500		1600		1	3700
5	5		110	2.785		2000		80		4	1100
6	6		50	0.2675		6000		80		2	450
5+6	5	6	80	2.785	0.2675	1000	3000	80	80	3	
7	7		310	1.5		2400		24		8	540
8	8		130	5.9		1300		340		4	800
9	9		200	0.9		2000		340		6	1300
8+9	8	9	165	5.9	0.9	650	1000	340	340	5	
10	10		5	0.04		15000		400		1	1300

*Annual inventory cost = 10% of item cost and one year = 240 - 8 hour days

**Normal distribution, coefficient of variance 0.1

Lost Sales Cost= 10% of item cost

Table 3. “The modified Bomberger Dataset”.

Option	Part N. Bomb		Set-up Costs			Unit Cost		Production Rate			Demand		Set-up Time		
	i		Ai	%Ai/Aj	%Ai/Aj	ui	%ui/uj	pi	%pi/pj	%pi/pij	di	%di/dj	ci	%ci/cj	%ci/cj
2	2		20	-33.33	25.00	0.1775	39.22	8000	-15.79	-50.00	400	-50.00	1	-50.00	50
3	3		30	50.00	-16.67	0.1275	-28.17	9500	18.75	-50.00	800	100.00	2	100.00	-25
	i	j	Aij			ui	uj	pij	pji		di	dj	cij		
2+3	2	3	25			0.1775	0.1275	4000	4750		400	800	2		
	i		Ai	%Ai/Aj	%Ai/Aj	ui	%ui/uj	pi	%pi/pj	%pi/pij	di	%di/dj	ci	%ci/cj	%ci/cj
5	5		110	120.00	-27.27	2.785	941.12	2000	-66.67	-50.00	80		4	100.00	-25
6	6		50	-54.55	60.00	0.2675	-90.39	6000	200.00	-50.00	80		2	-50.00	50
	i	j	Aij			ui	uj	pij	pji		di	dj	cij		
5+6	5	6	80			2.785	0.2675	1000	3000		80	80	3		
	i		Ai	%Ai/Aj	%Ai/Aj	ui	%ui/uj	pi	%pi/pj	%pi/pij	di	%di/dj	ci	%ci/cj	%ci/cj
8	8		130	-35.00	26.92	5.9	555.56	1300	-35.00	-50.00	340		4	-33.33	25
9	9		200	53.85	-17.50	0.9	-84.75	2000	53.85	-50.00	340		6	50.00	-16.67
	i	j	Aij			ui	uj	pij	pji		di	dj	cij		
8+9	8	9	165			5.9	0.9	650	1000		340	340	5		

Table 4. “Analysis of coproduction groups”.

However, item cost u_i is assumed to remain the same despite parts being grouped or not. In Table 4, the characteristics of the coproducts are analyzed. We examine the relationships among set-up costs, set-up time, unit cost, demand and

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production rate for the two products, i and j (2 and 3, 5 and 6, 8 and 9), which are candidates to be coproduced. For example in the first line, the value of -33.33% indicates that the set-up costs of product 2 are 33.33% lower than those of product 3. We also examine the relationships between set-up costs and set-up time, and the production rate between product i and its corresponding productive option, i+j. For example, the value of 25% in the first line indicates that the set-up costs of product 2 are 25% lower than those of the coproduction of 2+3.

We can observe that the set-up cost, unit cost, production rate, demand and set-up time values between the items inside a group are very different. We also note that the relationships between the coproduction or no coproduction values for each item are not that similar. So we can conclude that this kind of coproduction group is acceptable and that it can provide a good spectrum of different solutions depending on the scheduling rule.

We decided to include the lost sales cost as it indicates the service levels for fulfilled demand. We chose a modest value of 10% of the item cost, the same as the value of the holding costs. The demand rate shown in this table is when utilization is 88%.

6 Simulation results and analysis

In this section, we present three tables, Table 5, Table 6 and Table 7 which summarize the most important results of testing the three different scheduling rules (EMQ, Doll&Whybark and Fransoo). In order to decide the best productive option, we calculate the set-up, holding and lost sales costs for all the possible cases. Depending on the stock level of coproduct j, different stock strips may lead to the same result. It is consistent with the expected results, since some stock strips are included in others, i.e. if the stock level is in the stock strip B $\left[0,\left(Smax_j^* + ss_j^*\right)/2\right]$, it is going to pertain as well to stock strip C $\left[0,3\left(Smax_j^* + ss_j^*\right)/2\right]$ y D $\left[0,Smax_j^*\right]$. Hence, we will present the more representative cases, avoinding such repetitive featires, to clarify coproduction behavior. The cases are presented in the tables according to the decrease in the total costs. In each stated case, we indicate in the corresponding grip if the coproduction of the pair of products (2+3, 5+6 and 8+9) took place, or not, with the letter Y (Yes) or N

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(No), and if the stock strip of coproduction is done with the letters A,B,C,D, according to Table 2. For example, if we analyze the best result (Table 5-case 12) for case number 6 indicates that the coproduction of 5 with 6 is allowed, while the coproduction stock is inside stock strip A. So, the conditions of this coproduction are:

- If the run-out sequence indicates that the production of product 5 will start, product 6 will also be produced if its level of stock, I_6, is inside stock strip A, that is, $[0, ss_6]$.
- If the run-out sequence indicates that the production of product 6 will start, product 5 will also be produced if its level of stock, I_5, is inside stock strip A, that is, $[0, ss_5]$.

Figure 1, Figure 2, Figure 3 show the inventory of products 2,3,5,6,8 and 9 which are subject to the different situations and scheduling rules. There are six graphs in each figure. The situation of no coproduction is shown to the left of the figure in section (a), while the best coproduction option is shown to the right in section (b) for each group of coproducts. Finally, the best coproduction solution for all the heuristics is compared in Table 8.

6.1 Modified EMQ method

The total costs obtained with the EMQ and Doll and Whybark scheduling methods without coproduction are very close with 32315.40 and 31325.12 monetary units, respectively (Table 5: case 22 and Table 6: case 22).

If we compare the best result (case 22) result with the no coproduction option (case 4), we see that this coproduction type considerably decreases lost sales and set-up costs, despite having more holding costs.

In Table 5, we observe that there are many coproduction options (cases 5 to 22) with less total costs than the no coproduction option (case 4). Since it is a (s,S) policy, both values should be considered to analyse the result. According to the characteristics of products 2 and 3, its values for s and S are similar. Since they are similar, coproduction of 2&3 is giving fairly good results. Products 8 and 9, have also similar values for parameters s,S, but with minor differences. Accordingly

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the coproduction system works fairly well but with narrower stock strips, thus limiting the number of coproduction runs. Finally, coproduction for products 5 and 6, does not perform properly, and it might be due to the fact that their s and S values are not similar.

Case	Setup	Holding	Lost Sales	Total Cost	Coproduction					
					2+3	Stock Strip	5+6	Stock Strip	8+9	Stock Strip
1	27670	591.10	27743.92	56005.02	Y	B	N		N	
2	27600	563.68	27702.64	55866.32	Y	B	Y	A	N	
3	27850	515.22	27289.82	55655.04	N		Y	B	N	
4	27820	535.17	27122.64	55477.81	N		N		N	
5	27450	554.94	27467.29	55472.23	Y	A	N		N	
6	27450	544.07	27417.72	55411.80	Y	A	Y	A	N	
7	27820	527.68	26997.32	55345.01	N		Y	A	N	
8	23285	805.98	28124.22	52215.20	Y	C	Y	A	Y	A
9	23255	783.90	27794.16	51833.06	Y	D	Y	A	Y	A
10	23790	772.48	25428.98	49991.46	Y	C	Y	A	N	
11	23715	744.19	25146.87	49606.06	Y	C	N		N	
12	22485	741.68	24972.78	48199.46	Y	C	N		Y	A
13	18315	1048.81	18866.53	38230.33	N		Y	C	Y	A
14	16870	1307.07	15939.42	34116.49	Y	B	Y	A	Y	A
15	16785	1303.06	15881.93	33970.00	N		Y	B	Y	A
16	16945	1350.25	15668.66	33963.92	Y	B	N		Y	A
17	18255	885.49	14431.42	33571.91	Y	A	N		Y	A
18	16475	1377.53	15351.35	33203.87	N		Y	D	Y	A
19	16710	1340.01	15081.94	33131.95	N		N		Y	B
20	18015	881.36	13657.31	32553.67	Y	A	Y	A	Y	A
21	18070	948.70	13329.08	32347.78	N		N		Y	A
22	18040	948.24	13327.16	32315.40	N		Y	A	Y	A

Table 5. "Costs for different coproduction cases when applying the modified EMQ method".

When trying to test what happens with combined coproduction options, the three pairs simultaneously, results show that the system does only perform properly if the stock strips are narrow for both of the pairs but nor for all of them, as if the system was avoiding complex situations.

Figure 1 shows the stock of the coproducts at the simulation time. We observe the stock behavior of products 2 and 3 remains very similar in case 4 and case 25. In a given case product 3, reaches its Smax level, since the coproduction system together with the situation of product 2 stock level allows him to reach it. The behavior of stock levels for products 5 and 6, and 8 and 9 appears to be modified. The system behavior when coproducing either 5 and 6 or 8 and 9 allows to increase the quality of the response to sale losses. As it can be observed maximum

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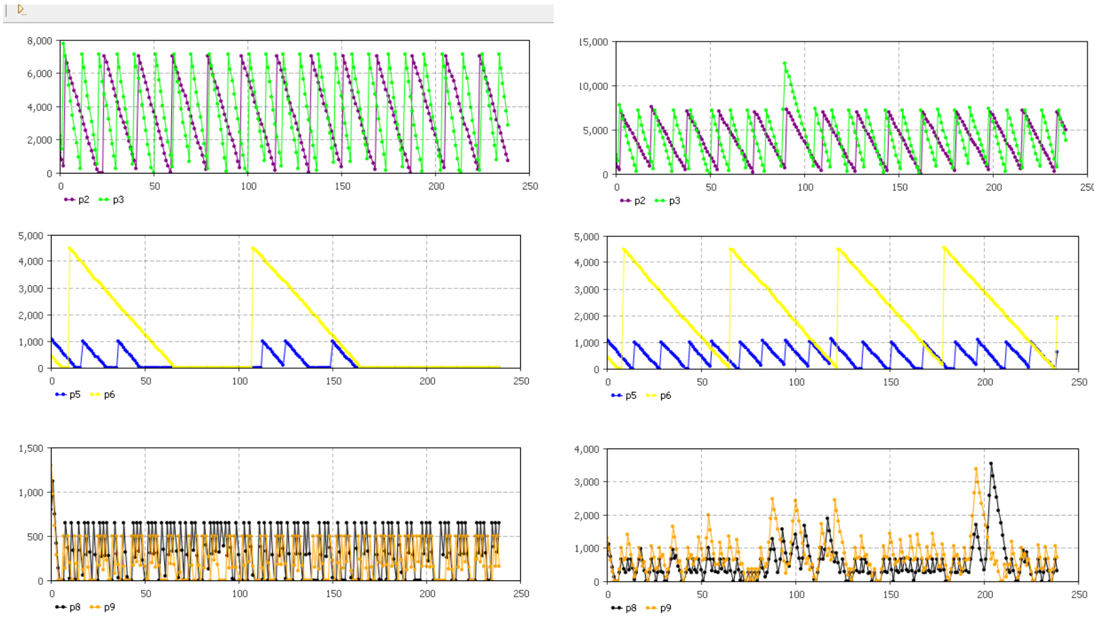
stock levels of the corresponding products are affected to this situation. This should be due to the production rate of product 9 being 53% higher than the production rate of product 8, while both their demand rates are equal (see Table 4).



(a) Items: 2&3, 5&6, 8&9 without coproduction – case 4

(b) Items: 2&3 without coproduction; 5&6, 8&9 with coproduction, Stock Strip A - case 22

Figure 1. “Impact of Coproduction addition to stocks in the modified EMQ rule”.

6.2 The modified Doll and Whybark method

Results obtained with this method (Table 6) are very similar to those obtained with the previously analyzed policy. This is so because both are (s, S) policies. Differences might be found since cycle times are evaluated in a different way. Results are shown in Table 1. The policy of Doll and Whybark uses a refined method to compute T, so that the results achieved are better as a whole. In this case, the profitability of coproduction is not as clear as in the previous case. In the case of coproduction of each pair in isolation, the results are equivalent to those of the previous section.

It is worthy to note that when coproduction is considered for more than one pair, one of them has to have a narrow stock strip (16, 13, 8). In this heuristic the best option is to coproduce the three pairs all together but with narrow stock strips. Case 22 (case 20 in the previous one) is the best, showing that both policies (this one and the previous one) are pretty similar.

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Case	Setup	Holding	Lost Sales	Total Cost	Coproduction					
					2+3	Stock Strip	5+6	Stock Strip	8+9	Stock Strip
1	27920	498.03	27582.83	56000.87	N		Y	B	N	
2	27765	46493	27475.27	55705.20	Y	B	N		N	
3	27890	506.69	27195.65	55592.35	N		Y	A	N	
4	27745	429.91	27398.31	55573.21	Y	B	Y	A	N	
5	27655	464.32	26331.91	54451.23	Y	A	N		N	
6	27460	539.05	26033.25	54032.30	N		N		N	
7	27415	456.60	26038.57	53910.17	Y	A	Y	A	N	
8	24350	688.85	26771.59	51810.44	Y	C	Y	A	N	
9	23525	618.98	27415.12	51559.10	Y	C	N		Y	A
10	22920	743.65	26207.21	49870.86	Y	C	Y	A	Y	A
11	24115	675.74	25056.88	49847.62	Y	C	N		N	
12	22675	689.54	26108.11	49472.65	Y	D	Y	A	Y	A
13	17875	1044.29	16867.03	35786.32	N		Y	D	Y	A
14	17810	1053.26	16801.40	35664.66	Y	B	N		Y	A
15	17635	1024.69	16785.75	35445.44	Y	B	Y	A	Y	A
16	17540	1110.35	16659.33	35309.67	N		Y	C	Y	A
17	17770	1104.61	16430.99	35305.61	N		N		Y	B
18	17410	1093.89	16403.35	34907.24	N		Y	B	Y	A
19	17505	860.69	14244.47	32610.16	Y	A	N		Y	A
20	17385	842.09	13953.34	32180.43	N		Y	A	Y	A
21	17490	855.03	13308.10	31653.13	N		N		Y	A
22	17300	812.58	13212.54	31325.12	Y	A	Y	A	Y	A

Table 6. "Costs for different coproduction cases by applying the modified Doll and Whybarkmethod".

Figure 2 shows the stock of the coproducts at the simulation time in the case 6 (without coproduction) and case 22 (best coproduction option). We observe that in case 22 products 2 and 3 are coproduced in some situations but its stock behavior remain very similar to case 6. Products 5 and 6 modify its behavior on case 22, since they are frequently coproduced, thus reducing cycle time of 6 and lost sales costs for both together. Stock levels for porducts 8 and 9, when coproduction, has major changes. Many sale losses are recovered, moreover, stock levels are above its original stock levels, far away from the stock out situation.

6.3 The modified Fransoo method

The results of the Fransoo method are very different from the results of other rules. In the Fransoo method, lost sales are lower at the expense of much higher holding costs (Table 7) yet the production runs are longer, hence the number of set-ups required is less. This method performs reasonably for coproduction (cases 9 to 20). However, the total cost obtained with the Fransoo method is much higher

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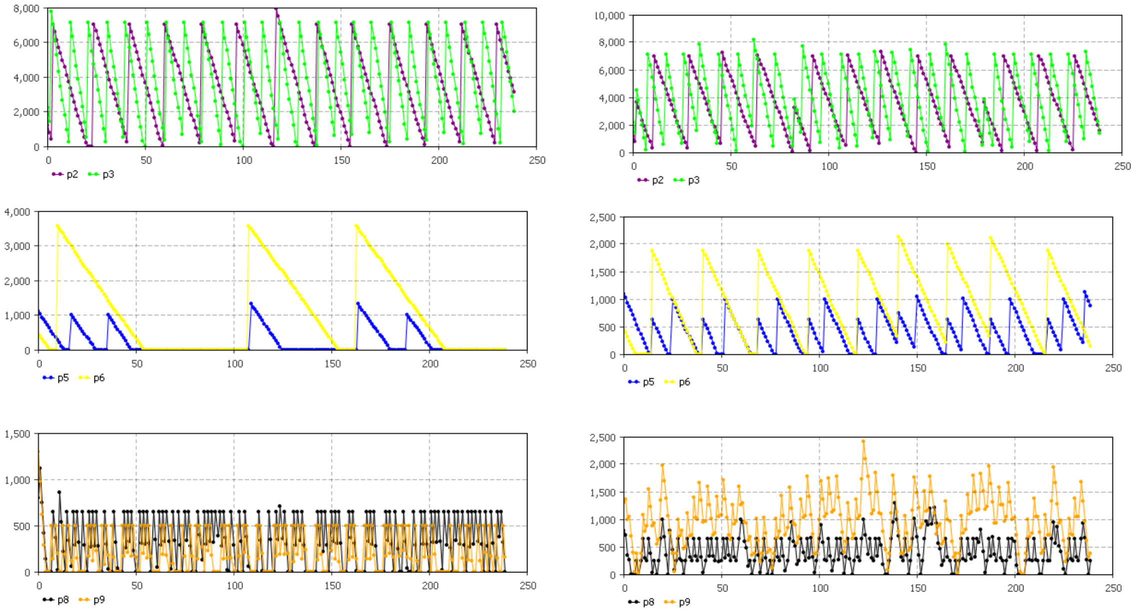
than with the other two methods in all cases. This could be because there are no constraints that interrupt the production of another item when one item starts to run out.



(a) Items: 2&3, 5&6, 8&9 without coproduction – case 6

(b) Items: 2&3, 5&6, 8&9 with coproduction, Stock Strip A - case 22

Figure 2. “Impact of Coproduction addition on stocks in the modified Doll and Whybark rule”.

Fransoo method, except for the first 9 cases of the table (1:9), outperforms well beyond the results obtained by the other two heuristics. The obtained setup costs are much lower, yet the costs of lost sales are radically different. With this method storage costs are slightly higher. A sensible explanation is as follows: being a system with high utilization rate (about 88%) and initial stocks very tight (see table 3), the system naturally tends to fall in lost sales for all products. Fransoo policy produces a given product regardless of the status of inventory levels of other products. In this way, the stock levels might recover each of the products reduce the loss of sales. On the other hand, the other two heuristic methods are (s, S) policies, as the low stock level products are always below its minimum level, they require the system to be continuously changing product and seek to retrieve it.

Results of Table 7 show that the cases 1 to 19, scenarios that consider more than one pair of co-products, are worse than the situation of non-coproduction (case 20). When two products are coproduced, maximum stock are reduced according to

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the combined cycle. This will slow the recovery of the stock for those products, so the total costs worsen (case 1, 5, 8, 10, 11, 15, 16, 18). With coproduction of only one pair of products in narrow stock strips they can be obtained, in some cases, good results (case 21:23).

Case	Setup	Holding	Lost Sales	Total Cost	Coproduction					
					2+3	Stock Strip	5+6	Stock Strip	8+9	Stock Strip
1	17835	1463.66	16804.71	36103.38	Y	C	N		Y	A
2	17720	1511.45	16791.88	36023.33	Y	D	Y	A	Y	A
3	17560	1504.35	16819.06	35883.40	Y	C	Y	A	Y	A
4	16860	1604.03	15556.87	34020.90	N		N		Y	B
5	16730	1675.12	15536.81	33941.93	N		Y	C	Y	A
6	16535	1704.15	15648.91	33888.05	N		Y	D	Y	A
7	16555	1704.27	15530.62	33789.89	N		Y	B	Y	A
8	16615	1587.17	15556.15	33758.32	Y	B	N		Y	A
9	16325	1710.10	15597.12	33632.22	Y	B	Y	A	Y	A
10	11075	4318.87	2472.07	17865.94	Y	D	Y	A	N	
11	11140	4229.60	2489.72	17859.32	Y	C	Y	A	N	
12	11140	4204.80	2485.81	17830.61	Y	C	N		N	
13	10200	4241.58	1724.06	16165.64	Y	B	N		N	
14	10155	4293.69	1698.83	16147.52	Y	A	N		N	
15	9995	4404.51	1703.73	16103.24	Y	B	Y	A	N	
16	10070	4322.71	1684.33	16077.03	Y	A	Y	A	N	
17	10090	4408.38	1532.82	16031.20	N		Y	B	N	
18	10330	4152.38	1535.90	16018.27	N		Y	A	Y	A
19	10165	4309.14	1532.82	16006.96	N		Y	A	N	
20	10145	4299.48	1557.99	16002.47	N		N		N	
21	9990	4323.64	1532.82	15846.46	N		Y	C	N	
22	10105	4144.10	1575.18	15824.28	N		N		Y	A
23	9810	4371.56	1532.82	15714.38	N		Y	D	N	
24	9915	4119.03	1601.36	15635.39	Y	A	N		Y	A
25	10095	4178.91	1055.68	15329.58	Y	A	Y	A	Y	A

Table 6. “Costs for different cases of coproduction by applying Fransoo modified method”.

The best situation is achieved with coproduction in narrow stock strips of the three products. A fair explanation of this effect is that within these bands coproduction is more limited, resulting in the optimal case that combines the benefits of co-production with those of isolated production. Notably, despite diversity between this heuristic and the other two heuristics, both best combinations of co-products are very similar.

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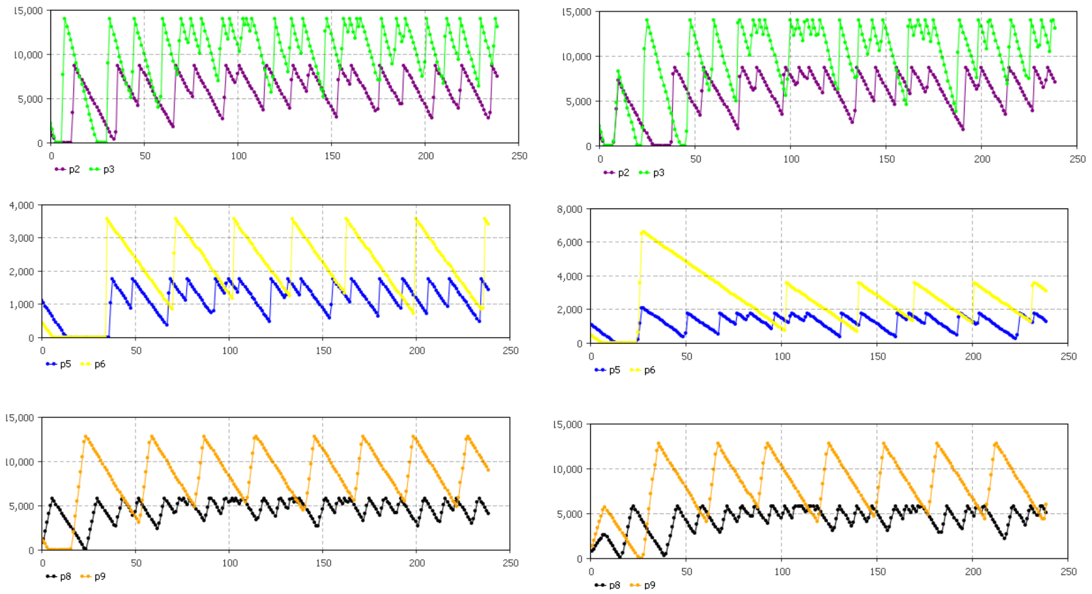

(a) Items: 2&3, 5&6, 8&9 without coproduction – case 20

(b) Items: 2&3, 5&6, 8&9 with coproduction, Stock Strip A - case 25

Figure 3. "Impact of Coproduction addition on the stocks in the Fransoo scheduling rule".

In this case according to Figure 3, the optimal situation of coproduction (case 25) evolves differently for each pair of coproducts. Thus, products 2, 3, 8 and 9 are greatly coproduced at the beginning of the horizon, but at the end of it (when the stock levels grow) there are not so many coproduction runs. But 5 and 6 are coproduced regularly during the whole run period.

Finally in Table 8, all the costs for the best productive options are shown. Here we see they all correspond to different coproduction options. As it has been above stated, the lower total costs are obtained with the Fransoo heuristic. With this rule, the set-up costs and lost sales than in the EMQ and Doll and Whybark rules, although holding cost is higher.

Heuristic	Setup	Holding	Lost Sales	Total Cost	Coproduction					
					2+3	Stock Strip	5+6	Stock Strip	8+9	Stock Strip
EMQ	18040	948.24	13327.16	32315.40	N		Y	A	Y	A
Doll&Whybark	17300	812.58	13212.54	31325.12	Y	A	Y	A	Y	A
Fransoo	10095	4178.91	1055.68	15329.58	Y	A	Y	A	Y	A

Table 7. "Cost Results of the three scheduling policies".

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7 Conclusions and future research

We aim to propose and simulate different scheduling policies with a new ELSP variant: ELSP with Coproduction. This problem occurs in an ELSP environment in which two products can be produced at a time on the same machine. To be able to draw conclusions, a simulation model was developed and results were obtained by employing modified Bomberger data which include items that could be produced simultaneously. To this end, this paper compares three simpler rules which were modified to consider coproduction. These heuristics are: EMQ, Doll and Whybark (1973) and Fransoo (1993).

The three simulated heuristics perform better under specific coproduction conditions than in the situation in which coproduction is not allowed. Indeed, there are eighteen scenarios in the modified EMQ heuristics, sixteen in the Doll and Whybark rule, and five in the Fransoo heuristics, whose total coproduction system costs are lower than the costs of the scenario without coproduction; see the section on the simulation results for further details. Therefore, we can affirm that coproduction is presented as an option to cut production system costs.

The best coproduction option in terms of costs has similar behaviour for all the heuristics. We observe that the best option of the heuristics decreases set-up costs and lost sales cost despite having more holding costs. If we consider the simulated case where we assume that set-up costs and the production rate are reduced by half when items are produced simultaneously, then coproduction appears an alternative with less set-ups and more inventory. It is important to point stress that this particular case corresponds to a facility whose utilization is 88%.

We may also observe that the scenarios for all the heuristics with a narrow coproduction stock strip perform better than scenarios with either a wide or a very wide coproduction stock strip. In other words, according to our definitions of stock strips (Table 2), when the rule employed for defining the sequence order indicates that a product has to be produced, and that this product could be produced along with another product, coproduction is always adequate provided the stock level of the second product is below half the sum of its maximum stock and its safety stock. When the coproduction stock strip is very wide, coproduction almost always takes place. So, we conclude that coproduction has to be deliberated and controlled

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whenever possible to achieve better results for coproduction rules because otherwise, holding costs and lost sales increase uncontrollably.

For all the experiments done, Fransoo rule (1993) appears to be the most appropriate scheduling rules for the system with coproduction. This result is quite interesting since most of the papers already published are considering rather high initial stock levels and thus they might reach quick stability with rules that overreact in front of many simultaneous stockouts leading to high sale losses.

Also, if we observe the values of the system for the three rules with no coproduction, we conclude that the behavior of the total costs of those rules which do not contemplate coproduction are the same as behavior of those with coproduction. So, it seems three rules adapt adequately to the coproduction phenomenon. It is also important to note that the reduction of the total costs achieved with coproduction in EMQ and Doll and Whybark rules is generally higher than the reduction achieved with the Fransoo rule.

In order to obtain better results, we can improve the way of choosing whether the product is to be produced separately or with another product by testing other heuristics or changing the way to calculate the target cycle using the common cycle policy. The coproduction of more than two products, or the coproduction in facilities using two stages would be other interesting areas to investigate.

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